

The $B \rightarrow \pi K$ puzzle and its relation to rare B and K decays

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Abstract. The standard-model interpretation of the ratios of charged and neutral $B \rightarrow \pi K$ rates, R_c and R_n , respectively, points towards a puzzling picture. Since these observables are affected significantly by colour-allowed electroweak (EW) penguins, this “ $B \rightarrow \pi K$ puzzle” could be a manifestation of new physics in the EW penguin sector. Performing the analysis in the R_n – R_c plane, which is very suitable for monitoring various effects, we demonstrate that we may, in fact, move straightforwardly to the experimental region in this plane through an enhancement of the relevant EW penguin parameter q . We derive analytical bounds for q in terms of a quantity L , which measures the violation of the Lipkin sum rule, and point out that strong phases around 90° are favoured by the data, in contrast to QCD factorisation. The $B \rightarrow \pi K$ modes imply a correlation between q and the angle γ that, in the limit of negligible rescattering effects and colour-suppressed EW penguins, depends only on the value of L . Concentrating on a minimal flavour-violating new-physics scenario with enhanced Z^0 penguins, we find that the current experimental values on $B \rightarrow X_s \mu^+ \mu^-$ require roughly $L \leq 1.8$. As the $B \rightarrow \pi K$ data give $L = 5.7 \pm 2.4$, L has either to move to smaller values once the $B \rightarrow \pi K$ data improve or new sources of flavour and CP violation are needed. In turn, the enhanced values of L seen in the $B \rightarrow \pi K$ data could be accompanied by enhanced branching ratios for the rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$, $B \rightarrow X_s \nu \bar{\nu}$ and $B_{s,d} \rightarrow \mu^+ \mu^-$. Most interesting turns out to be the correlation between the $B \rightarrow \pi K$ modes and $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, with the latter depending approximately on a single “scaling” variable $\bar{L} = L \cdot (|V_{ub}/V_{cb}|/0.086)^{2,3}$.

1 Introduction

The rich physics potential of $B \rightarrow \pi K$ modes is attracting a lot of interest in the B -physics community [1]. Decays of this kind are caused by $\bar{b} \rightarrow \bar{d}d\bar{s}$, $\bar{u}u\bar{s}$ quark-level processes, and receive contributions from penguin and tree topologies, where the latter are associated with the angle γ of the unitarity triangle of the Cabibbo–Kobayashi–Maskawa (CKM) matrix. Since the CKM factor $|V_{us}V_{ub}^*/(V_{ts}V_{tb}^*)| \approx 0.02$ is tiny, $B \rightarrow \pi K$ modes are governed by QCD penguins. Moreover, we have also contributions from electroweak (EW) penguins. In the case of $B_d^0 \rightarrow \pi^- K^+$ and $B^+ \rightarrow \pi^+ K^0$ modes, these topologies are colour suppressed and play hence only a minor rôle. On the other hand, EW penguins contribute also in colour-allowed form to $B^+ \rightarrow \pi^0 K^+$ and $B_d^0 \rightarrow \pi^0 K^0$. Consequently, they are expected to be sizable in these modes, i.e. of the same order of magnitude as the tree topologies. Interference between the tree and penguin topologies leads to sensitivity on γ .

The isospin flavour symmetry of strong interactions suggests one to consider the following combinations of $B \rightarrow \pi K$ decays: the “mixed” $B_d \rightarrow \pi^\mp K^\pm$, $B^\pm \rightarrow \pi^\pm K$ system [2–5], the charged $B^\pm \rightarrow \pi^0 K^\pm$, $B^\pm \rightarrow \pi^\pm K$ system [6–9], and the neutral $B_d \rightarrow \pi^\mp K^\pm$, $B_d \rightarrow \pi^0 K$ system [8, 9].

The CP -conserving and CP -violating observables of each system provide sufficient information to determine γ and a corresponding strong phase. For the following discussion, we use the ratios of the CP -averaged $B \rightarrow \pi K$ branching ratios introduced in [8]:

$$R \equiv \left[\frac{\text{BR}(B_d^0 \rightarrow \pi^- K^+) + \text{BR}(\overline{B}_d^0 \rightarrow \pi^+ K^-)}{\text{BR}(B^+ \rightarrow \pi^+ K^0) + \text{BR}(B^- \rightarrow \pi^- \overline{K}^0)} \right] \frac{\tau_{B^+}}{\tau_{B_d^0}}, \quad (1)$$

$$R_c \equiv 2 \left[\frac{\text{BR}(B^+ \rightarrow \pi^0 K^+) + \text{BR}(B^- \rightarrow \pi^0 K^-)}{\text{BR}(B^+ \rightarrow \pi^+ K^0) + \text{BR}(B^- \rightarrow \pi^- \overline{K}^0)} \right], \quad (2)$$

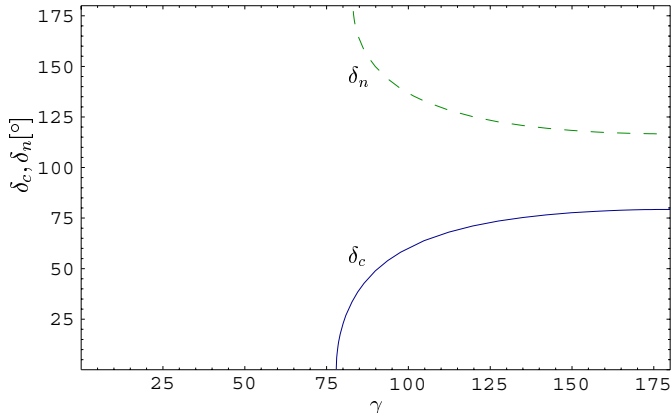
$$R_n \equiv \frac{1}{2} \left[\frac{\text{BR}(B_d^0 \rightarrow \pi^- K^+) + \text{BR}(\overline{B}_d^0 \rightarrow \pi^+ K^-)}{\text{BR}(B_d^0 \rightarrow \pi^0 K^0) + \text{BR}(\overline{B}_d^0 \rightarrow \pi^0 \overline{K}^0)} \right]. \quad (3)$$

In Table 1, we summarise the current experimental status of these observables. The final averages for $R_{(c,n)}$ given in this table have been obtained by using the average branching ratios from the data of CLEO [10], BaBar [11] and Belle [12] that read

$$\begin{aligned} \text{BR}(B^+ \rightarrow \pi^0 K^+) &= (12.82 \pm 1.08) \cdot 10^{-6}, \\ \text{BR}(B^+ \rightarrow \pi^+ K^0) &= (20.62 \pm 1.35) \cdot 10^{-6} \end{aligned} \quad (4)$$

Table 1. The current experimental status of the observables $R_{(c,n)}$

Observable	CLEO ('03)	BaBar ('03)	Belle ('03)	Average
R	1.04 ± 0.26	0.97 ± 0.11	0.91 ± 0.11	0.95 ± 0.07
R_c	1.37 ± 0.40	1.28 ± 0.20	1.16 ± 0.20	1.24 ± 0.13
R_n	0.70 ± 0.24	0.86 ± 0.15	0.73 ± 0.17	0.81 ± 0.10

**Fig. 1.** $|\delta_c|$ and $|\delta_n|$ as functions of γ for $R_c = 1.24$ and $R_n = 0.81$, respectively

and

$$\begin{aligned} \text{BR}(B_d^0 \rightarrow \pi^0 K^0) &= (11.21 \pm 1.36) \cdot 10^{-6}, \\ \text{BR}(B_d^0 \rightarrow \pi^- K^+) &= (18.16 \pm 0.79) \cdot 10^{-6}. \end{aligned} \quad (5)$$

Finally, we have used $\tau_{B^+}/\tau_{B_d^0} = 1.086 \pm 0.017$.

As was already emphasised by two of us in [9], the pattern of $R_c > 1$ and $R_n < 1$ is actually very puzzling within the standard model (SM). To understand the problem let us first note that R_c and R_n allow us to determine CP -conserving strong phases δ_c and δ_n as functions of γ , respectively. As can be seen in Fig. 1, the current central values for R_c and R_n imply lower bounds for γ around 80° , and very different values for the strong phases. However, these strong phases are not expected to differ so largely from each other, as can be seen from their exact definitions in [8]. This problem becomes also obvious in the contour plots in the $B \rightarrow \pi K$ observable space shown in [13]. On the other hand, no anomalous behaviour is indicated by the observable R of the mixed $B \rightarrow \pi K$ system, where EW penguins only contribute in colour-suppressed form. Consequently, as noted in [9], this puzzle could be a manifestation of new-physics contributions in the EW penguin sector, which is a rather popular scenario for physics beyond the SM to enter the $B \rightarrow \pi K$ system [14, 15]. This point was also very recently re-emphasised in [16–19].

In 2000, when [9] was written, the $B_d^0 \rightarrow \pi^0 K^0$ channel had just been observed by the CLEO collaboration. Now we have a much better experimental picture, where interestingly *all* three experiments point towards $R_c > 1$ and $R_n < 1$, as can be seen in Table 1, whereas $R \sim 1$. Although the experimental uncertainties are still sizable, we think that it is legitimate and interesting to return to this

puzzle and to explore in more detail whether enhanced EW penguins could really provide a solution. Another important element of our analysis are rare B and K decays. If we restrict ourselves to new-physics scenarios with “minimal flavour violation” (MFV) [20] and enhanced Z^0 penguins, we obtain a nice connection between the $B \rightarrow \pi K$ puzzle and $B \rightarrow X_s \mu^+ \mu^-$, $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$, $B \rightarrow X_s \nu \bar{\nu}$ and $B_{s,d} \rightarrow \mu^+ \mu^-$ decays. In order to make our findings more transparent, we shall neglect colour-suppressed EW penguins, $SU(3)$ -breaking contributions and rescattering effects. A more detailed analysis including these effects and addressing more technical aspects can be found in [21]. The outline of the present paper is as follows: in Sect. 2, we explore the impact of enhanced EW penguins on the observables R_c and R_n . In Sect. 3, we discuss the connection between the value of the relevant $B \rightarrow \pi K$ EW penguin parameter q and Z penguins in the restricted class of MFV models specified above. These results are then applied in Sect. 4 to analyse rare B and K decays and to explore the implications of the corresponding experimental constraints for the $B \rightarrow \pi K$ system. Finally, we summarise our conclusions in Sect. 5.

2 Enhanced EW penguins in $B \rightarrow \pi K$ decays

If we employ the parametrisation introduced in [8], we may write within the approximations stated above

$$R_{c,n} = 1 + 2r_{c,n} B \cos \delta_{c,n} + [B^2 + \sin^2 \gamma] r_{c,n}^2, \quad (6)$$

where

$$B \equiv q - \cos \gamma \quad (7)$$

is a “universal” quantity for the charged and the neutral $B \rightarrow \pi K$ systems. In addition to γ , it depends on a parameter q , which measures the ratio of the sum of the colour-allowed and colour-suppressed EW penguins with respect to the sum $T+C$ of the colour-allowed and colour-suppressed tree-diagram-like contributions. Using $SU(3)$ flavour-symmetry arguments, we can calculate the EW penguin parameter q within the SM as follows [6]:

$$q|_{\text{SM}} = 0.69 \times \left[\frac{0.086}{|V_{ub}/V_{cb}|} \right], \quad (8)$$

where $|V_{ub}/V_{cb}| = 0.086 \pm 0.008$. Here we have taken NLO corrections into account and used the most recent input parameters [22]. The strong phase ω associated with q vanishes in the $SU(3)$ limit, and we have already used $\omega = 0$ in (6). Even values of ω up to 20° have very little influence on our analysis (see [21] for a detailed discussion).

The parameters r_c and r_n describe, roughly speaking, the ratio of $T + C$ and penguin amplitudes, where the latter are determined by the CP -averaged $B^\pm \rightarrow \pi^\pm K$ and $B_d \rightarrow \pi K$ rates, respectively. Using the exact definitions given in [8] and taking into account that $|T + C|$ can be fixed through the $SU(3)$ flavour symmetry with the help of the CP -averaged $B^\pm \rightarrow \pi^\pm \pi^0$ rate [23], we arrive at

$$r_c = \sqrt{2} \left| \frac{V_{us}}{V_{ud}} \right| \frac{f_K}{f_\pi} \sqrt{\frac{\text{BR}(B^\pm \rightarrow \pi^\pm \pi^0)}{\text{BR}(B^\pm \rightarrow \pi^\pm K^0)}} = 0.201 \pm 0.017, \quad (9)$$

$$r_n = \left| \frac{V_{us}}{V_{ud}} \right| \frac{f_K}{f_\pi} \sqrt{\frac{\text{BR}(B^\pm \rightarrow \pi^\pm \pi^0)}{\text{BR}(B_d^0 \rightarrow \pi^0 K^0)}} \sqrt{\frac{\tau_{B_d^0}}{\tau_{B^+}}} = 0.185 \pm 0.018, \quad (10)$$

where we have used $\text{BR}(B^\pm \rightarrow \pi^\pm \pi^0) = (5.3 \pm 0.8) \cdot 10^{-6}$ and have taken factorizable $SU(3)$ -breaking corrections into account through the factor f_K/f_π .

Finally, δ_c and δ_n measure the strong phase differences between the tree amplitude $T + C$ and the $B^+ \rightarrow \pi^+ K^0$ and $B_d^0 \rightarrow \pi^0 K^0$ penguin amplitudes, respectively. As seen in (6), with $r_c \approx r_n \approx 0.2$ and γ and q being universal quantities, there is *no way* to reproduce $R_c = 1.24 \pm 0.13$ and $R_n = 0.81 \pm 0.10$ for the same values of δ_c and δ_n . This is in particular clear for the special case of $\delta_c \approx \delta_n \approx 0$ corresponding to QCD factorisation [19, 24], where one finds generally $R_c \simeq R_n > 1.0$. Even the inclusion of enhanced ‘‘charming penguins’’ [25] does not help, which points to a different solution to be discussed below.

In the spirit of Fig. 1, the charged and neutral $B \rightarrow \pi K$ systems were considered separately in [9], also in view of enhanced EW penguins. The new element we are using here is the relation

$$\delta_n = \delta_c + \varphi, \quad (11)$$

where

$$\sin \varphi = \frac{qr_c \sin \delta_c}{\sqrt{b}}, \quad \cos \varphi = \left[\frac{1 - qr_c \cos \delta_c}{\sqrt{b}} \right], \quad (12)$$

with

$$b \equiv \frac{R}{R_n} = \left(\frac{r_c}{r_n} \right)^2 = 2 \left[\frac{\text{BR}(B_d^0 \rightarrow \pi^0 K^0) + \text{BR}(\overline{B}_d^0 \rightarrow \pi^0 \overline{K}^0)}{\text{BR}(B^+ \rightarrow \pi^+ K^0) + \text{BR}(B^- \rightarrow \pi^- \overline{K}^0)} \right] \frac{\tau_{B^+}}{\tau_{B_d^0}} = 1.18 \pm 0.16, \quad (13)$$

providing a link between the charged and neutral $B \rightarrow \pi K$ systems. As discussed in detail in [21], these relations can be derived with the help of the general parameterisations introduced in [8]. The remarkable feature of (11) and (12) is that we may induce a difference between δ_c and δ_n through the EW penguin parameter q , provided δ_c is sizable. Using the expression on the left-hand side of (12), we obtain

$$\sin \varphi|_{\text{SM}} \lesssim qr_c \approx 0.14, \quad (14)$$

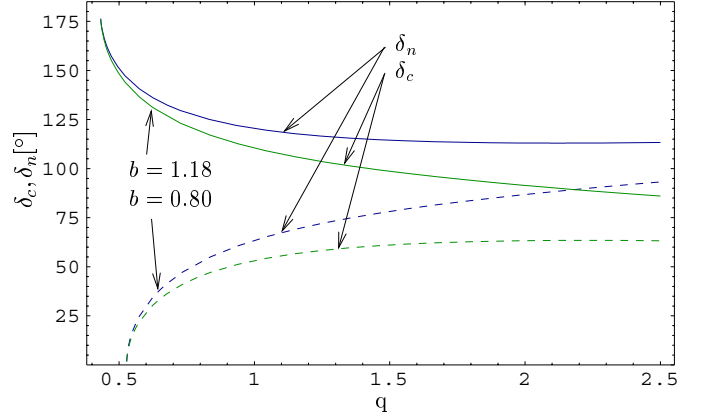


Fig. 2. $|\delta_c|$ and $|\delta_n|$ as functions of q for $r_c = 0.20$ and $b = 1.18$ (solid) and $b = 0.80$ (dashed)

corresponding to a phase difference φ of at most $\sim 8^\circ$ for $\delta_c = 90^\circ$ within the SM. This feature is the origin of the puzzle reflected by Fig. 1. However, we observe also that the phase shift is increased through an enhancement of the EW penguin parameter q . The burning question is now whether this mechanism can actually reproduce the experimental pattern of the observables $R_{c,n}$. Before we address this exciting issue, let us first note that the relations given in (12) imply, furthermore, the following expression:

$$\cos \delta_c = \frac{1 - b + q^2 r_c^2}{2qr_c}, \quad (15)$$

allowing us to calculate δ_c as a function of q for given values of b and r_c , which are fixed through experiment. In Fig. 2, the solid lines give δ_c and δ_n as functions of q for central values of b and r_c .

The variable b coincides with R_{00} in [19], and consequently $b = 0.79 \pm 0.08$ in the QCD factorisation approach [19, 24], which is significantly below the experimental value in (13). In Fig. 2, the dashed lines give δ_c and δ_n as functions of q for this low value of b . The smallness of b in the latter approach can be attributed to the *destructive* interference of QCD and EW penguin contributions in the $B_d^0 \rightarrow \pi^0 K^0$ decay that takes place when the relevant phase δ_n is small. Our finding in [9] and in Fig. 2 that $\delta_n > 90^\circ$ can be interpreted as a *constructive* interference of QCD and EW penguin contributions in the $B_d^0 \rightarrow \pi^0 K^0$ decay making the corresponding rate significantly larger than in the QCD factorisation approach.

If we now go back to (6), it is an easy exercise to derive the following expressions:

$$R_c = 1 + \frac{B}{q}(1 - b) + [B(B + q) + \sin^2 \gamma] r_c^2, \quad (16)$$

$$R_n = 1 + \frac{B}{bq}(1 - b) + [B(B - q) + \sin^2 \gamma] \frac{r_c^2}{b}. \quad (17)$$

Let us emphasise that (16) and (17) do *not* involve any CP -conserving strong phases. Since b and $r_{c,n}$ (up to non-factorizable $SU(3)$ -breaking corrections) can be directly determined from experiment, our two key observables R_c

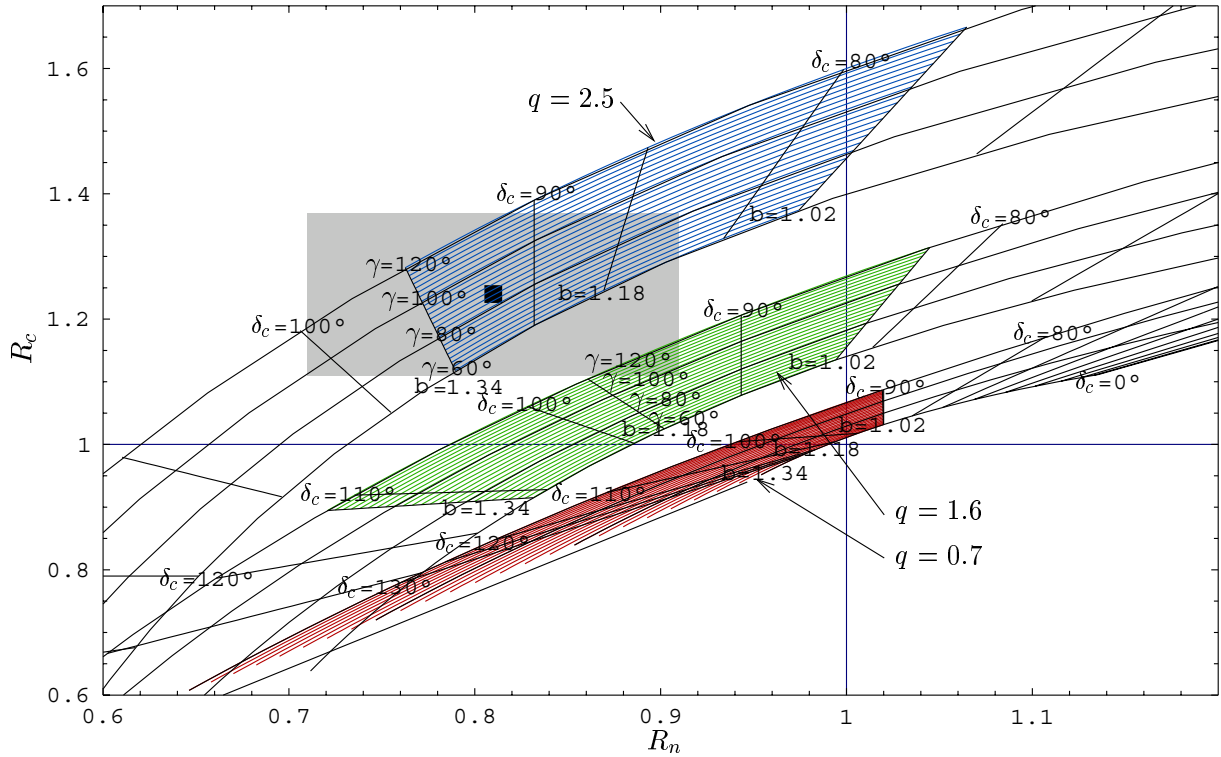


Fig. 3. Allowed regions in the R_n - R_c plane for $q = 0.7$ (SM), 1.6 and 2.5. The grey rectangle indicates the 1- σ experimental bounds on R_n and R_c with their central values

and R_n depend now only on the two “unknowns” q and γ . Consequently, we have sufficient information to determine these quantities, which through (11), (12) and (15) would then fix the strong phases $\delta_{c,n}$ as well. It is easy to see that (16) and (17) are invariant under the following transformations:

$$q \rightarrow -q \quad \text{and} \quad \gamma \rightarrow \pi - \gamma. \quad (18)$$

Consequently, for each solution (q, γ) of our problem there is a second one, which is given by $(-q, \pi - \gamma)$. Since q may, in principle, also be negative in the presence of new physics, we cannot discard this case. However, as we will see in Sect. 4, at least for MFV models, $q > 0$ turns out to be more interesting.

It is very instructive to consider the situation in the R_n - R_c plane, as shown in Fig. 3, where the 1- σ experimental ranges for R_c and R_n are indicated by the grey rectangle. Each of the broad bands in this plane represents a given value of q , while different lines within a band correspond to different values of the angle γ , which we vary between 60° and 120° . A specific position on a line fixed by q and γ corresponds to a particular value of b (as indicated on the inside of the bands) and at the same time (through (15)) to a particular value of δ_c (as indicated on the outside of the bands). The closely spaced lines are only drawn when b lies within the 1- σ experimental region (13), for b outside this region only the “skeleton” of the band is drawn.

We observe two remarkable features already advertised above.

(1) An increase of q brings us straightforwardly to the experimental region. This is, in fact, necessary for *any* value

of γ if current data are confirmed when precision improves. (2) A large strong phase δ_c around 90° and consequently also large δ_n are required.

The last feature indicates that the corrections to factorisation are significantly larger than estimated in the QCD factorisation approach [19,24]. Interestingly, evidence for a large strong phase $\delta_c \sim 90^\circ$ follows also from an analysis of CP violation in $B_d \rightarrow \pi^+\pi^-$ [13], where the external experimental sign of the corresponding direct CP asymmetry points, for $\gamma \in [0^\circ, 180^\circ]$, towards the interval $\delta_c \in [0^\circ, 180^\circ]$. In the decays employed in [13], EW penguins may only contribute in colour-suppressed form.

In order to obtain further insights, it is useful to exploit that (16) and (17) imply

$$L \equiv \frac{(R_c - 1) + b(1 - R_n)}{2r_c^2} = Bq, \quad (19)$$

where L can be determined from experiment (up to non-factorizable $SU(3)$ -breaking corrections entering through the parameter r_c). Taking into account (7), this quantity allows us to calculate q as a function of γ with the help of

$$q = \frac{1}{2} \left[\cos \gamma \pm \sqrt{\cos^2 \gamma + 4L} \right], \quad (20)$$

where the plus and minus signs give $q > 0$ and $q < 0$, respectively. We observe then the third remarkable feature. (3) Whereas (16) and (17) involve four experimental quantities R_n , R_c , b and r_c , the correlation between q and γ depends on a single quantity, the variable L .

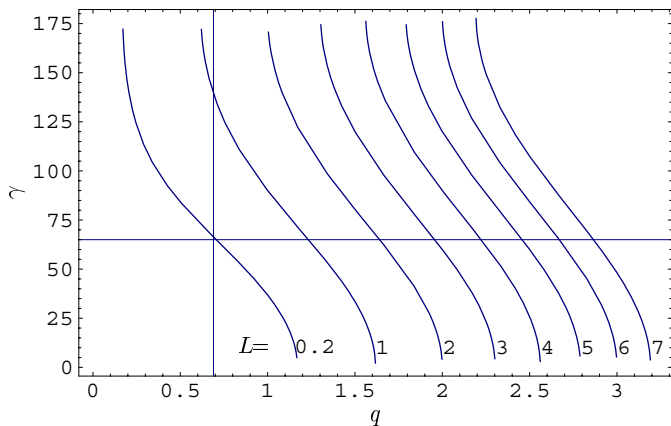


Fig. 4. γ as a function of q for different values of L . The central values of γ and q within the SM are also shown

In Fig. 4, we show γ as a function of q for different values of L . The interesting aspect of this correlation with respect to the rare decays sensitive to the CKM element $|V_{td}|$ is the strong decrease of the angle γ with increasing $q > 0$. As the decrease of γ is related to the decrease of $|V_{td}|$, this correlation has profound implications for the rare decay $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, as discussed in Sect. 4. For $q < 0$, $|q|$ increases with increasing γ .

If we vary γ between 0° and 180° , we obtain

$$\frac{1}{2} \left[-1 + \sqrt{1 + 4L} \right] \leq |q| \leq \frac{1}{2} \left[+1 + \sqrt{1 + 4L} \right], \quad (21)$$

providing interesting analytical bounds on q . Using the current experimental values given in Table 1, (9) and (13), we find

$$L = 5.7 \pm 2.4, \quad 1.4 \leq |q| \leq 3.4. \quad (22)$$

Consequently, the $B \rightarrow \pi K$ data favour values of $|q|$ which are substantially larger than the SM value in (8). The variable L measures, up to an overall factor, the violation of the Lipkin sum rule [26]. Indeed, using the definition of L in (19), we find

$$L = \left\{ 2[\Gamma(B^\pm \rightarrow \pi^0 K^\pm) + \Gamma(B_d \rightarrow \pi^0 K)] - [\Gamma(B^\pm \rightarrow \pi^\pm K) + \Gamma(B_d \rightarrow \pi^\mp K^\pm)] \right\} / \{2r_c^2 \Gamma(B^\pm \rightarrow \pi^\pm K)\}. \quad (23)$$

As can be seen in (19), the large value for L implied by the $B \rightarrow \pi K$ data is directly related to $R_c > 1$ and $R_n < 1$, i.e. to the $B \rightarrow \pi K$ puzzle already pointed out in [9]. The simple expression for L in (19) implies also

$$L|_{\text{SM}} \sim 0.18, \quad (24)$$

where we have used the SM values of $q = 0.69$ and $\gamma = 65^\circ$. The possible large violation of the Lipkin sum rule and theoretical interpretations were also discussed in [17, 27] and very recently in [19]. In Sect. 4, we will see that L provides an interesting link between the $B \rightarrow \pi K$ puzzle and rare B and K decays.

As we already mentioned after formulae (16) and (17), the four quantities q , γ , δ_c and δ_n can be determined within the approximations used in this paper. The formulae (11), (12), (15), (16) and (17) are the basis for this determination and have been used in the plot in Fig. 3. Still it is instructive to discuss the determination of q and γ in more detail. To this end, we use (7) and (19) to find

$$\cos \gamma = q - \frac{L}{q}, \quad (25)$$

which allows us to eliminate γ in the expression (17) for R_n , thereby yielding

$$q^2 = U \pm \sqrt{U^2 - V}, \quad (26)$$

with

$$U = \frac{b(1 - R_n) + (1 + L)r_c^2}{2r_c^2}, \quad V = \frac{(b - 1)L}{r_c^2}. \quad (27)$$

Formulae (25) and (26) then give the analytic expressions for γ and q , respectively.

This discussion shows that the whole system is rather constrained. Consequently, the fact that a simple change of q provides a solution to the $B \rightarrow \pi K$ puzzle is non-trivial. As is already evident from Fig. 3, if the parameter b would substantially differ from the experimental value in (13), for instance if it was smaller than 1.0,¹ we would miss the experimental values of R_n and R_c even with increased q , although we could well accommodate the violation of the Lipkin sum rule. This feature is also reflected by the fact that (26) may not provide a solution at all. However, with the current experimental uncertainties, no discrepancy emerges and we consider it very non-trivial to be able to move to the experimental region in the R_n - R_c plane by just increasing the value of q .

Another important consequence of our analysis are potentially large values of the pseudo-asymmetries introduced in [8], which may be written – within the approximations employed above – as follows:

$$A_0^{(c,n)} = 2r_{c,n} \sin \delta_{c,n} \sin \gamma. \quad (28)$$

In the case of large CP -conserving phases $\delta_{c,n}$ as indicated by our numerical studies, these asymmetries could be as large as 0.3. A similar pattern emerges if one assumes enhanced “charming penguin” contributions [25]. In view of very large experimental uncertainties in $A_0^{(c,n)}$, such large values of $A_0^{(c,n)}$ cannot be ruled out at present. We will briefly return to this point in Sect. 4. Finally, the enhanced EW penguins would enhance other branching ratios, like the ones for $B_s \rightarrow \pi^0 \phi$ and $B_s \rightarrow \rho^0 \phi$ [28].

To summarise, if the current data will be confirmed with increasing experimental precision, there are four messages from these considerations.

¹ For these considerations it is important that there are four independent observables, corresponding to four branching ratios; thus, b is indeed independent of R_n and R_c

- (1) The EW penguin parameter q must be substantially larger than in the SM for any value of γ .
- (2) Both δ_c and δ_n must be large and must differ significantly from each other, where the difference is correlated with the value of q as given in (12).
- (3) The value of b must be larger than 1 in order for the enhanced value of q to provide a solution to the $R_c > 1$ and $R_n < 1$ puzzle. Moreover we see the following.
- (4) A correlation between q and γ is implied by the $B \rightarrow \pi K$ data, as can be seen in (20). It depends on the single variable L , which measures the violation of the Lipkin sum rule.

3 The value of q and the Z^0 penguin

Next we want to investigate whether the enhanced values of q and L given in (22) are compatible with the measured branching ratios of rare decays in a specific extension of the SM, where a simple relation between the parameter q entering the $B \rightarrow \pi K$ observables and the Z^0 penguin diagram function C [29], which governs many rare decays, can be established.

To this end, we consider a simple extension of the SM, where the dominant new-physics contributions enter only the Z^0 penguin function C , which depends in the SM only on the ratio m_t^2/M_W^2 and equals $C \approx 0.80$. For this value of C , the q given in (8) is obtained. This class of extensions of the SM has already been discussed in several papers in the past [30–32], but not in the context of $B \rightarrow \pi K$ decays. They can be considered as a restricted class of MFV models [20] in which the CKM matrix is the only source of flavour and CP violation, the local operators are as in the SM and the restriction comes from the assumption that the dominant new-physics effects enter through the Z^0 penguin diagrams.

The value of q can be determined from the Wilson coefficients $C_9(\mu_b)$ and $C_{10}(\mu_b)$ ($\mu_b = \mathcal{O}(m_b)$) of the $(V - A) \otimes (V - A)$ EW penguin operators Q_9 and Q_{10} entering the effective Hamiltonian for $\Delta B = 1$ non-leptonic decays [29]. Explicitly, in the $SU(3)$ flavour limit, one has [6]

$$qe^{i\omega} = -\frac{3}{2} \frac{1}{\lambda |V_{ub}/V_{cb}|} \left[\frac{C_9(\mu_b) + C_{10}(\mu_b)}{C'_1(\mu_b) + C'_2(\mu_b)} \right], \quad (29)$$

where, following [8], we have replaced the Wilson coefficients $C_{1,2}$ of the current–current operators $Q_{1,2}$ present in the formulae of [6] by

$$\begin{aligned} C'_1(\mu_b) &= C_1(\mu_b) + \frac{3}{2} C_9(\mu_b), \\ C'_2(\mu_b) &= C_2(\mu_b) + \frac{3}{2} C_{10}(\mu_b), \end{aligned} \quad (30)$$

as should be done in the case of enhanced EW penguins. We observe that we have $\omega = 0$ in this approximation.

The coefficients $C_9(\mu_b)$ and $C_{10}(\mu_b)$ can be calculated by means of NLO renormalisation group equations from the initial conditions for the Wilson coefficients at $\mu = \mathcal{O}(m_t)$ entering the Hamiltonian in question. The function

C , which appears in these initial conditions along with box and other penguin contributions, depends on the gauge of the W propagator, but this dependence enters only in the subleading terms in m_t^2/M_W^2 and is cancelled by the one of the box diagrams.

As we have seen above, the current data for the $B \rightarrow \pi K$ decays favour an increased value of q independently of γ with respect to the SM estimate, and this implies also a higher value of the Z^0 penguin function C . Performing the full NLO renormalisation group analysis by means of the formulae in [33], and assuming that only the function C is affected by new physics, we find the following approximate but accurate expression for the dependence of C on q :

$$C(\bar{q}) = 2.35 \bar{q} - 0.82, \quad q = \bar{q} \left[\frac{0.086}{|V_{ub}/V_{cb}|} \right], \quad (31)$$

where we have introduced \bar{q} in order to separate the C dependence in q from the $|V_{ub}/V_{cb}|$ dependence. To our knowledge, this relation appears in the literature for the first time. On the other hand, in [15], the impact of rather involved new-physics scenarios that go beyond the MFV framework on the EW parameters in the $B \rightarrow \pi K$ system has been investigated. However, the authors of [15] did not simultaneously discuss the correlation with rare K and B decays. This is the next topic we want to discuss.

4 The rare decays

The function $C(\bar{q})$ is an important ingredient in any analysis of rare semi-leptonic K and B decays. Even if QCD corrections to Z^0 penguins in non-leptonic decays differ from those in the case of semi-leptonic decays, an explicit two-loop calculation [34] shows that the difference in these corrections is very small and it is a very good approximation to use the same Z^0 penguin function in non-leptonic and semi-leptonic decays. Moreover, these differences appear only at the NNLO level in non-leptonic decays [34], which is clearly beyond the scope of this paper.

The present best upper bound on the function C follows from the data on $B \rightarrow X_s l^+ l^-$. A recent update of [32] given in [18] implies that the maximal enhancement of C over the SM value cannot be larger than 2–3; that is, $C \leq 2.0$. Our own analysis of $B \rightarrow X_s l^+ l^-$ indicates that indeed $C > 2.0$ (for $C > 0$) and $|C| > 2.4$ (for $C < 0$) are very improbable as they give a $\text{BR}(B \rightarrow X_s l^+ l^-)$ which is by more than a factor of 2 larger than the average of the Belle and BaBar data [35]. In view of substantial experimental and theoretical errors in the full branching ratio, it would be premature to attach a confidence level to these findings but in what follows we will assume that

$$\begin{aligned} |C| &\leq \begin{cases} 2.0 & C > 0, \\ 2.4 & C < 0, \end{cases} & |\bar{q}| &\leq \begin{cases} 1.20 & \bar{q} > 0, \\ 0.67 & \bar{q} < 0, \end{cases} \\ |q| &\leq \begin{cases} 1.47 & q > 0, \\ 0.82 & q < 0, \end{cases} \end{aligned} \quad (32)$$

where we have used (31) to obtain the bounds on $|\bar{q}|$, and have then conservatively chosen $|V_{ub}/V_{cb}| \geq 0.070$ to obtain the bounds on $|q|$. These bounds can be refined in the

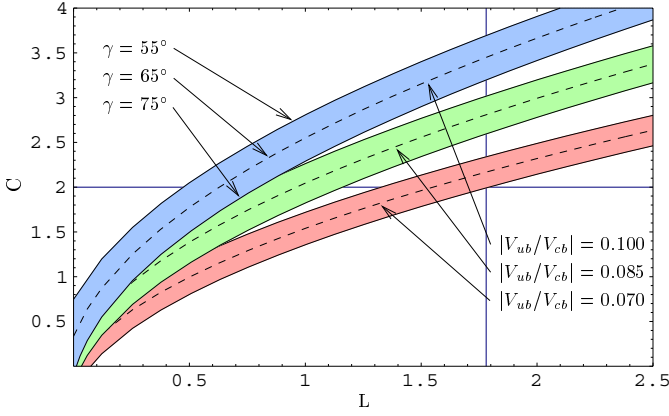


Fig. 5. C as a function of L for $|V_{ub}/V_{cb}| = 0.070, 0.085, 0.100$. The different lines in each band correspond to different $\gamma = (65 \pm 10)^\circ$. The upper bound for L in (33) is obtained for $\gamma = 75^\circ$ and $|V_{ub}/V_{cb}| = 0.070$

future. Finally, taking $\gamma = (65 \pm 10)^\circ$, as required by MFV models [36], we find with the help of (19)

$$L \leq \begin{cases} 1.78 & q > 0, \\ 1.14 & q < 0. \end{cases} \quad (33)$$

In Fig. 5, we show C in the case of $C > 0$ as a function of L for $\gamma = (65 \pm 10)^\circ$ and various values of $|V_{ub}/V_{cb}|$. The different lines in the $|V_{ub}/V_{cb}|$ bands correspond to different values of γ . This plot establishes the connection between $B \rightarrow \pi K$ decays and rare K and B decays in the class of simple models considered here. This connection will become more precise when the determinations of γ and $|V_{ub}/V_{cb}|$ improve.

The allowed values for L in (33) are clearly outside the $1\text{-}\sigma$ range for L in (22) and more than a factor of 3 below the central values of L . Moreover, for $\gamma = (65 \pm 10)^\circ$, the allowed range for $|q|$ from $B \rightarrow K\pi$ decays with L given in (22) is $1.6 \leq |q| \leq 3.1$. Consequently, in the context of the simple new-physics scenario considered here, the enhancement of EW penguins implied by the $B \rightarrow \pi K$ data appears to be too strong to be consistent with the data on $\text{BR}(B \rightarrow X_s l^+ l^-)$, unless values for L outside the range (22) – but still higher than the SM value $L \approx 0.2$ – are considered. If this really turned out to be the case, the enhanced L implied by the $B \rightarrow \pi K$ data should be accompanied by an enhanced $\text{BR}(B \rightarrow X_s l^+ l^-)$ close to the upper limit coming from the Belle and BaBar data. Similarly, the enhanced value of L should be accompanied by an enhanced forward-backward asymmetry in this decay that increases with increasing $C(\bar{q})$.

In this spirit we will now analyse the $K \rightarrow \pi\nu\bar{\nu}$ and $K_L \rightarrow \pi^0 e^+ e^-$ decays with the vision that, if $B \rightarrow \pi K$ decays indeed signal an enhancement of L and consequently of C , this enhancement could eventually be tested through these rare decays.

The branching ratios for $K \rightarrow \pi\nu\bar{\nu}$ and $K_L \rightarrow \pi^0 e^+ e^-$ are usually written in terms of the functions X and Y [29], respectively. These functions are linear combinations of the C function and the $\Delta F = 1$ box diagram function

that we assume to take the SM value, $B_{\text{SM}} = -0.182$ for $m_t(m_t) = 167$ GeV. To our knowledge, this assumption is satisfied in all MFV models that have been considered in the literature.

We find then

$$X(\bar{q}) = 2.35 \bar{q} - 0.09, \quad Y(\bar{q}) = 2.35 \bar{q} - 0.64, \quad (34)$$

and consequently, using (32),

$$|X| \leq \begin{cases} 2.73 & X > 0, \\ 1.66 & X < 0, \end{cases} \quad |Y| \leq \begin{cases} 2.18 & Y > 0, \\ 2.21 & Y < 0, \end{cases} \quad (35)$$

to be compared with $X = 1.53 \pm 0.04$ and $Y = 0.98 \pm 0.04$ in the SM.

Inserting $X(\bar{q})$ and $Y(\bar{q})$ in the known expressions for the branching ratios of various rare decays [29], it is straightforward to calculate these branching ratios for a given q and L considered in the context of $B \rightarrow \pi K$ decays. Moreover, the correlation between q and the angle γ in (20) will also have some impact on the rare decays sensitive to V_{td} whereas it has no impact on decays sensitive to V_{ts} . These correlations between the new physics in $B \rightarrow \pi K$ and rare K and B decays are discussed in more detail in [21]. Below we discuss only selected aspects of this analysis.

We consider first the decays $K \rightarrow \pi\nu\bar{\nu}$ for which the branching ratios are given as follows [37]:

$$\begin{aligned} \text{BR}(K^+ \rightarrow \pi^+ \nu\bar{\nu}) \\ = 4.75 \cdot 10^{-11} \cdot \left[(\text{Im } F_t)^2 + (\text{Re } F_c + \text{Re } F_t)^2 \right], \end{aligned} \quad (36)$$

$$\text{BR}(K_L \rightarrow \pi^0 \nu\bar{\nu}) = 2.08 \cdot 10^{-10} \cdot (\text{Im } F_t)^2, \quad (37)$$

where

$$F_c = \frac{\lambda_c}{\lambda} P_0(X), \quad F_t = \frac{\lambda_t}{\lambda^5} X(\bar{q}). \quad (38)$$

Here $\lambda_i = V_{is}^* V_{id}$, whereas $P_0(X) = 0.39 \pm 0.06$ results from the internal charm contribution [38], which is assumed not to be affected by new physics.

Note that for a given value of the angle γ , and the values of $|V_{cb}|$ and $|V_{ub}/V_{cb}| \geq 0.070$, the CKM factors λ_i can be calculated, and consequently we can study the branching ratios in question as a function of γ and L . The $|V_{ub}/V_{cb}|$ dependence in the relation of C to L shown in Fig. 5 and the correlation between $C(\bar{q})$ and γ implied by (20) have to be consistently taken into account in this analysis.

In Fig. 6a, we show $\text{BR}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$ as a function of L for $\gamma = (65 \pm 10)^\circ$. The horizontal line represents the 68% C.L. upper bound following from the AGS E787 collaboration result [39]

$$\text{BR}(K^+ \rightarrow \pi^+ \nu\bar{\nu}) = (15.7_{-8.2}^{+17.5}) \cdot 10^{-11}. \quad (39)$$

The sensitivity of $\text{BR}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$ to γ seen in Fig. 6a is substantially smaller than in the usual SM analysis. This feature is due to the correlation between q and γ in (20), and the correlation between C and γ for fixed L in Fig. 5: the variations of γ and $C > 0$ in $\text{BR}(K^+ \rightarrow \pi^+ \nu\bar{\nu})$ compensate each other to a large extent.

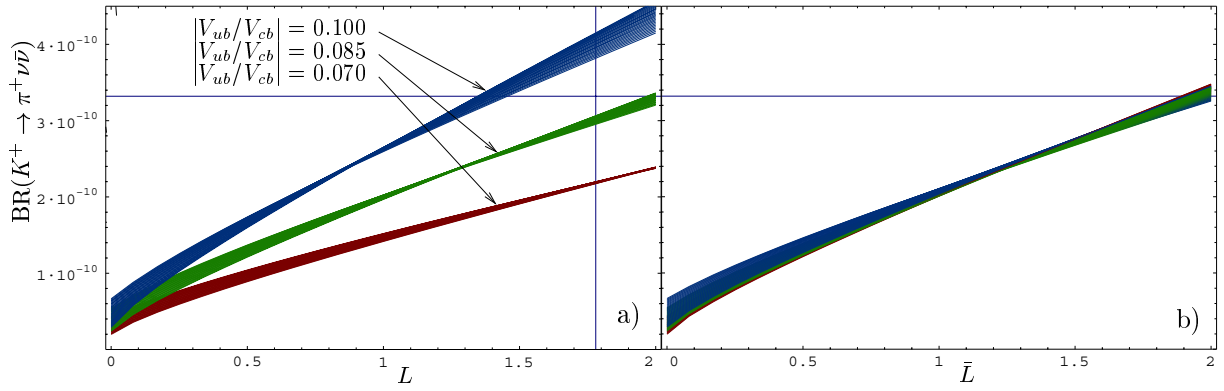


Fig. 6a,b. $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ as a function of L and \bar{L} for $\gamma = (65 \pm 10)^\circ$

On the other hand, we observe a strong sensitivity of $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ obtained in this manner on $|V_{ub}/V_{cb}|$, which is essentially not present in the usual analysis. This fact originates in the correlation between C and L in Fig. 5 that depends on $|V_{ub}/V_{cb}|$. However, as demonstrated in Fig. 6b, this dependence in $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ can be summarised to a good approximation by introducing the following “scaling” variable:

$$\bar{L} \equiv L \left(\frac{|V_{ub}/V_{cb}|}{0.086} \right)^{2.3}. \quad (40)$$

Then $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ depends to a good approximation only on \bar{L} , with a weak residual dependence on $|V_{ub}/V_{cb}|$ and γ . Needless to say, the knowledge of $|V_{ub}/V_{cb}|$ is essential for the usefulness of the correlation between $B \rightarrow \pi K$ and rare decays discussed here.

The bound $L \leq 1.8$ required by the $\text{BR}(B \rightarrow X_s \mu^+ \mu^-)$ data translates into $\bar{L} \leq 1.1$ and consequently values for $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ as high as $25 \cdot 10^{-11}$ are allowed. However, even for $0.5 \leq \bar{L} \leq 1.0$, $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ can be a factor of 2 larger than the SM prediction $(7.7 \pm 1.2) \cdot 10^{-11}$ [37, 40], and close to the central values of the AGS E787 experiment. We also note that the present data on $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ put a much weaker constraint on L than $B \rightarrow X_s \mu^+ \mu^-$, but this can change in the future as the $K^+ \rightarrow \pi^+ \nu \bar{\nu}$ decay is theoretically cleaner. Since the experimental situation for $K_L \rightarrow \pi^0 \nu \bar{\nu}$ is not as satisfactory, we mention only that for $\bar{q} = 1.20$ this decay is enhanced roughly by a factor of 3 with respect to its SM value.

Another interesting process is the rare decay $K_L \rightarrow \pi^0 e^+ e^-$ reconsidered recently within the SM [41] in view of new NA48 data on $K_S \rightarrow \pi^0 e^+ e^-$ and $K_L \rightarrow \pi^0 \gamma \gamma$ [42] that allow a much better evaluation of the indirectly CP -violating and CP -conserving contributions to $\text{BR}(K_L \rightarrow \pi^0 e^+ e^-)$. In order to illustrate the implications of the enhanced Z^0 penguins on this ratio we set, in the spirit of [30], all remaining loop functions at their SM values and keep only the function C as a free parameter. Setting moreover all other parameters of [41] at their central values, we find

$$\begin{aligned} \text{BR}(K_L \rightarrow \pi^0 e^+ e^-) & \quad (41) \\ &= 10^{-12} [18.3 + 12.5 \bar{y}_{7V} + 4.4(\bar{y}_{7V}^2 + \bar{y}_{7A}^2)], \end{aligned}$$

where

$$\bar{y}_{7V} = 0.56 + 0.69Y(\bar{q}) - 0.64C(\bar{q}), \quad \bar{y}_{7A} = -0.69Y(\bar{q}). \quad (42)$$

In the SM, $\bar{y}_{7V} = 0.73$ and $\bar{y}_{7A} = -0.68$ at the NLO level [43]. Note that \bar{y}_{7V} depends only very weakly on \bar{q} .

For $\text{BR}(K_L \rightarrow \pi^0 e^+ e^-)$, we find then with $\bar{q}_{\text{max}} = 1.20$ the central value $4.1 \cdot 10^{-11}$ to be compared with the central value $3.2 \cdot 10^{-11}$ within the SM [41] and the experimental upper bound from KTeV [44]: $2.8 \cdot 10^{-10}$ (90% C.L.). Even for $\bar{q} = 2.5$ one finds $9.1 \cdot 10^{-11}$, which is still compatible with the data. Consequently, this decay does not offer useful bounds on q and L at present.

Next, we would like to comment briefly on some other decays that, while not yet observed at the level required to further tighten the bounds, do exhibit rather striking effects: these are the decays $B \rightarrow X_s \nu \bar{\nu}$ and $B_{s,d} \rightarrow \mu^+ \mu^-$, where the values in (35) correspond to enhancements over the SM estimates of the branching ratios by factors of 3.2 and 5.0, respectively. This gives roughly $\text{BR}(B \rightarrow X_s \nu \bar{\nu}) = 1 \cdot 10^{-4}$, $\text{BR}(B_s \rightarrow \mu^+ \mu^-) = 2 \cdot 10^{-8}$ and $\text{BR}(B_d \rightarrow \mu^+ \mu^-) = 5 \cdot 10^{-10}$, all compatible with the existing upper bounds on these processes. Observation of these modes at this level could signal the presence of enhanced Z^0 penguins.

Finally, we can now ask what is the impact of the rare decays on the $R_n - R_c$ plane in Fig. 3. In Fig. 7, we show the area close to (1, 1) in the $R_n - R_c$ plane and plot the contours for $q = 0.7$ and $q = 1.3$ (as consistent with (32)) and $\gamma = (65 \pm 10)^\circ$. The comments about how variations of q , b and δ_c are denoted made in the context of Fig. 3 still apply.

We observe that a modest shift of the experimentally allowed region brings the values of R_n , R_c and b into agreement with the scenario that employs an enhanced C ($q = 1.3$). A larger shift (corresponding to shifts of each of the four $B \rightarrow K\pi$ branching ratios into the respective preferred direction by 1–1.6 σ) will even move the experimental region towards the contour that only employs the SM value for q . In both of these cases, however, the strong phases have to be close to 90° , implying large direct CP asymmetries and contradicting QCD factorisation. As can be seen clearly from Fig. 7, small values of δ_c corresponding to small direct CP asymmetries can only be obtained

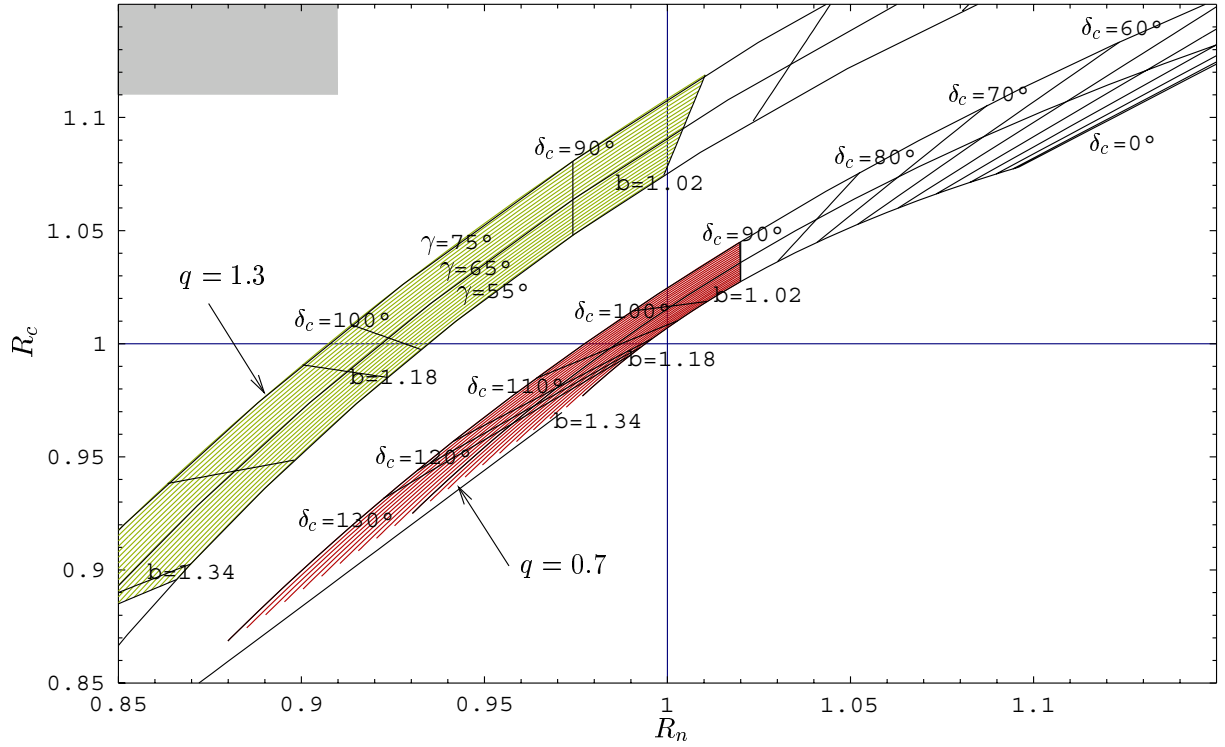


Fig. 7. Allowed regions in the R_n – R_c plane for $q = 0.7$ and $q = 1.3$. The grey rectangle represents the lower right-hand corner of the experimental $1\text{-}\sigma$ region that can be fully seen in Fig. 3

for $R_n \gtrsim 1.1$, which is currently consistently disfavoured by all three experiments. Small asymmetries are also obtained for $R_c, R_n < 1$, but in this case $\delta_c, \delta_n \approx 180^\circ$ (c.f. Fig. 2).

5 Conclusions

The current data on $B \rightarrow \pi K$ decays cannot be easily explained within the standard model. Extending the 2000 analysis of two of us, we have demonstrated that the present data on these decays can be correctly described, provided we have the following.

- (1) The EW penguin parameter q is by a factor of 2–4 larger than its SM value;
- (2) the strong phases δ_c and δ_n are large and in particular $\cos \delta_n$ is negative.

These findings are true for any value of the angle γ so that changing only γ does not solve the problem if the former two conditions are not satisfied.

- (3) Consequently, the present data indicate that the corrections to factorisation are significantly larger than estimated in the QCD factorisation approach and, moreover, new-physics contributions may be signalled.

Using the general parametrisation of $B \rightarrow \pi K$ decays proposed in [8], we have derived a number of new relations with the help of the $SU(3)$ flavour symmetry, taking factorizable $SU(3)$ -breaking corrections into account. In particular we see the following.

- (1) The $B \rightarrow \pi K$ data imply a correlation between q and γ

which depends on a single variable L . This quantity measures the violation of the Lipkin sum rule and can be determined experimentally. In the case of $q > 0$, an increase of γ decreases the q required to fit the data. Moreover, q increases with increasing L .

- (2) The CP -conserving strong phase difference $\delta_n - \delta_c$ is correlated with q and δ_c , and increases with q and $\sin \delta_c$.
- (3) The measurement of the four CP -averaged $B \rightarrow \pi K$ branching ratios allows us to determine q , γ , δ_c and δ_n .
- (4) Consequently, the fact that an increase of q allows us to obtain straightforwardly an agreement with the data should be considered as very non-trivial.
- (5) We have proposed to monitor these correlations in the form of allowed regions in the R_n – R_c plane.

Concentrating then on a MFV new-physics scenario with enhanced Z^0 penguins, we have derived relations between the parameter q relevant for $B \rightarrow \pi K$ decays and the functions C , X and Y that enter the branching ratios for rare K and B decays. This allowed us to analyse the correlations between the latter decays and the $B \rightarrow \pi K$ system with the following findings.

- (1) In the context of the simple new-physics scenario considered here, the enhancement of EW penguins implied by the $B \rightarrow \pi K$ data appears to be too strong to be consistent with the data on $\text{BR}(B \rightarrow X_s l^+ l^-)$: whereas the latter decays imply $L \leq 1.8$, the $B \rightarrow \pi K$ data requires $L = 5.7 \pm 2.4$. Consequently, either L has to move to smaller values once the $B \rightarrow \pi K$ data improve, or new sources of flavour and CP violation are needed.
- (2) Enhanced values of L , while still smaller than those calculated from the present $B \rightarrow \pi K$ data, could be ac-

accompanied by enhanced branching ratios for the rare decays $K^+ \rightarrow \pi^+ \nu \bar{\nu}$, $K_L \rightarrow \pi^0 e^+ e^-$, $B \rightarrow X_s \nu \bar{\nu}$ and $B_{s,d} \rightarrow \mu^+ \mu^-$.

(3) In particular, we have found a correlation between the $B \rightarrow \pi K$ modes and $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$, with the latter depending approximately only on a single “scaling” variable $\bar{L} = L \cdot (|V_{ub}/V_{cb}|/0.086)^{2-3}$. For $L \approx 1.0$, an enhancement of $\text{BR}(K^+ \rightarrow \pi^+ \nu \bar{\nu})$ by a factor of 2 with respect to the SM is expected.

In order to describe our results in a transparent manner, we have neglected rescattering effects, colour-suppressed EW penguins, and the strong phase ω , which enters the EW penguin parameter q and is predicted to vanish in the $SU(3)$ limit. These effects are incorporated in the general parametrisation presented in [8] and will be discussed in detail in [21]: we find that rescattering effects have a minor impact and cannot explain the $B \rightarrow \pi K$ puzzle. The inclusion of colour-suppressed EW penguins makes this puzzle slightly more pronounced, i.e. the required value of q increases. Larger effects could emerge from a non-vanishing value of ω , as already discussed in [8,9], but as long as $\omega \leq 20^\circ$, also these effects are small. It should be recalled that a non-vanishing ω could come only from non-factorizable $SU(3)$ -breaking effects and a value substantially larger than 20° appears to be very unlikely.

It will be very exciting in the next couple of years to follow the development of the values of $R_{c,n}$, R , $r_{c,n}$, b , q and $\delta_{c,n}$, and to monitor the allowed regions in the R_n – R_c plane. Equally interesting will be the correlation of the $B \rightarrow \pi K$ data with those for rare B and K decays for which more accurate measurements should soon be available.

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